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# Global homeomorphism and applications to the existence and uniqueness of solutions of some differential equations

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Jiangsu, 213002, P.R. China**Abstract**

In this paper, the global diffeomorphism theorem due to Zampieri was generalized. The concept of the basin of attraction is the main tool of our exposition in discussing the diffeomorphism between Banach spaces. The existence and uniqueness of a solution of the fourth order elliptic boundary value problem was proved by employing our generalized theorem. The results of this paper generalize some known theorems.

**MSC:** 35J40**Keywords:** Banach space; diffeomorphism; boundary value problem; the basin of attraction

## 1 Introduction

An important problem in nonlinear analysis is to find precise conditions for a local diffeomorphism  $f$  between two Banach spaces  $E, F$  to be a global one. Many authors have investigated this problem and have obtained results along this line, see [1–4]. The most famous conclusion among these may be the Hadamard-Levy theorem [2], it states that a continuously differentiable map  $f$  between two Banach spaces  $E$  and  $F$ , which is a local diffeomorphism and satisfies the condition

$$\int_0^\infty \frac{ds}{\omega(s)} = \infty, \quad \| [f'(x)]^{-1} \| \leq \omega(\|x\|),$$

is a homeomorphism, where  $\omega : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is a continuous increasing mapping.

In 1974, Plastock [3] reduced the global homeomorphism problem to one of finding the conditions for a local homeomorphism to satisfy the ‘line lifting property’. This property was then shown to be equivalent to a limiting condition which was designated by condition (L).

**Definition 1.1** (condition (L)) Let  $D$  an open connected set of  $E$ , the continuous mapping  $f : D \subseteq E \rightarrow F$  satisfies condition (L) if for every continuous function  $q : [0, 1] \rightarrow F$  with  $q(0) \in f(D)$ , and for every  $x_0 \in f^{-1}(0)$ , there exists a continuous function  $p : [0, b) \rightarrow D$  with  $p(0) = x_0$  such that  $f(p(t)) = q(t)$ ,  $0 \leq t < b$ , then there exists a sequence  $t_n \rightarrow b$  as

$n \rightarrow \infty$  such that

$$\lim_{n \rightarrow \infty} p(t_n)$$

exists and is in  $D$ .

The powerful generalization of the Hadamard-Levy theorem is as follows.

**Theorem 1.1** *Let  $f : E \rightarrow F$  be a local homeomorphism. Then  $f$  is a global homeomorphism of  $E$  onto  $F$  if and only if  $f$  satisfies condition (L).*

Theorem 1.1 was employed to prove the global diffeomorphism and the existence and uniqueness of solution of boundary value problems by some authors [3–6].

Let us consider about the Cauchy problem:

$$\dot{x}(t) = G(x(t)), \quad x(0) = x_0$$

with

$$G(x) := -f'(x)^{-1}(f(x) - f(x_0)),$$

which is equivalent to

$$f(x(t)) - f(x_0) = e^{-t}(f(x) - f(x_0)).$$

In [7], Shen found that the condition (L) could be replaced with  $x(t)$  being defined on  $[0, +\infty)$ . Then the set  $A$  of all the  $x$  according to  $x(t)$  is called the attraction basin of  $x_0$ , which means  $x$  will be attracted to  $x_0$  as  $t \rightarrow +\infty$ . Thus, Shen presented Theorem 1.2.

**Lemma 1.1** [8] *Let  $x_0 \in D$ . Then for any  $x \in D$ , the path-lifting problem*

$$\begin{cases} f(\gamma_x(t)) = f(x_0) + e^{-t}(f(x) - f(x_0)), & t \in R, \\ \gamma_x(0) = x, & \gamma_x(t) \in D \end{cases}$$

*has a unique continuous solution  $t \rightarrow \gamma_x(t)$  defined on the maximal open interval  $I_x = (t_x^-, t_x^+)$ ,  $-\infty \leq t_x^- < 0 < t_x^+ \leq +\infty$ . Moreover, the set  $\{(x, t) \in D \times R : t \in I_x\}$  is open in  $D \times R$  and the mapping  $(x, t) \rightarrow \gamma_x(t)$  is continuous.*

**Theorem 1.2** *Let  $f : E \rightarrow F$  be a local homeomorphism. Then  $f$  is a global homeomorphism of  $E$  onto  $F$  if and only if  $\gamma_x(t)$  is defined on  $R$  for all  $x \in A$ , namely,  $\gamma_x(t)$  can also be extended to  $-\infty$ . Here  $A = \{x \in D : t_x^+ = +\infty\}$  is the basin of attraction of  $x_0$ .*

In 1992, using the nonnegative auxiliary scalar coercive function, that is, continuous mappings  $k : E \rightarrow R^+$  with  $k(x) \rightarrow +\infty$  as  $\|x\| \rightarrow +\infty$ , Zampieri [9] also generalized the Hadamard-Levy theorem to show the following situation.

**Theorem 1.3** [9] *The mapping  $f \in C^1(E; F)$  is a global diffeomorphism onto  $F$  if*

- (1)  $f'(x) \in \text{Isom}(E, F) \forall x \in E$ ;
- (2)  $\sup_{\|x\| \leq r} \|f'(x)^{-1}\| < +\infty \forall r : 0 < r < +\infty$ ;

- (3) *there exists a locally Lipschitzian coercive function  $k : E \rightarrow R^+$  which admits all  $D_V^+ k(x)$ , and it is such that  $\sup\{D_V^+ k(x) : v = f'(x)^{-1}u, x \in E, u \in F, \|u\| = 1\} < +\infty$ .*

The purpose of this paper is to generalize the above Zampieri [9] theorems by making use of Theorem 1.2 and the attraction basin. In our main Theorem 2.2, in Section 2, we replaced Zampieri's condition (3) by the conditions:

(3) *there exists a coercive function  $k : E \rightarrow R^+$  which admits the directional right derivatives  $D_V^+ k(x)$ , for arbitrary  $x, x_0 \in E$ ,  $y = f(x) \in F$ ,  $v = f'(x)^{-1}(y - f(x_0))$  and a continuous mapping  $h : R^+ \rightarrow R^+$  such that  $D_V^+ k(x) \leq h(k(x))$  and for any  $r_0 > 0$ , the maximum solution of the initial value problem*

$$\begin{cases} r' = h(r), \\ r(0) = r_0 \end{cases}$$

is defined on  $[0, a)$  and there exists a sequence  $t_n \rightarrow a$  as  $n \rightarrow \infty$  such that  $\lim_{n \rightarrow \infty} r(t_n) = r^*$  is finite.

Thus Theorem 1.3 is a corollary of our theorem. Our proofs are different from the proofs given by Zampieri [9]. In Section 3, we will apply our theorems to the existence and uniqueness of solutions of the fourth order elliptic boundary value problem.

## 2 The main theorems

In this paper, we denote  $E, F$ , Banach spaces,  $D$  is an open subset of  $E$ . The following theorem will be employed to prove our main theorems.

**Theorem 2.1** [10] *Let  $U(t, u)$  be continuous on an open  $(t, x)$ -set  $H$  and  $u = u^0$  the maximal solution of  $u'(t) = U(t, u)$ ,  $u(0) = u_0$ . Let  $v(t)$  be a continuous function on  $[t_0, t_0 + b]$  satisfying the condition  $v(t_0) \leq u_0$ ,  $(t, v(t)) \in H$ , and  $v(t)$  has a derivative  $v'(t)$  on  $t_0 \leq t \leq t_0 + b$  such that*

$$v'(t) \leq U(t, v(t)).$$

*Then, on a common interval of existence of  $u^0(t)$  and  $v(t)$*

$$v(t) \leq u^0(t). \quad (2.1)$$

Now, we will show our main theorems.

**Theorem 2.2** *Let  $f : D \subset E \rightarrow F$  be a  $C^1$  mapping, if*

- (1)  $f'(x) \in \text{Isom}(E, F) \forall x \in E$ ;
- (2)  $\sup_{\|x\| \leq r} \|f'(x)^{-1}\| < +\infty \forall r : 0 < r < +\infty$ ;
- (3) *there exists a coercive function  $k : E \rightarrow R^+$  which admits the directional right derivatives  $D_V^+ k(x)$  for arbitrary  $x, x_0 \in E$ ,  $y = f(x) \in F$ ,  $v = f'(x)^{-1}(y - f(x_0))$  and a continuous mapping  $h : R^+ \rightarrow R^+$  such that  $D_V^+ k(x) \leq h(k(x))$  and for any  $r_0 > 0$ , the maximum solution of the initial value problem*

$$\begin{cases} r' = h(r), \\ r(0) = r_0 \end{cases} \quad (2.2)$$

is defined on  $[0, a)$  and there exists a sequence  $t_n \rightarrow a$  as  $n \rightarrow \infty$  such that  $\lim_{n \rightarrow \infty} r(t_n) = r^*$  is finite. Then  $f$  is a global homeomorphism of  $D$  onto  $F$ .

*Proof* Since  $f'(x) \in \text{Isom}(E, F) \forall x \in E, f'(x)^{-1}$  exists for all  $x \in D$ . Hence  $f$  is a local homeomorphism of  $D$ . Then, in the light of Theorem 1.2, we only need show that for all  $x \in A$ ,  $\gamma_x(t)$  can also be extended to  $-\infty$ . Namely, we need to consider the problem in the opposite direction.

Consider the path-lifting problem of the mapping  $f$

$$\begin{cases} f(\gamma_x(t)) = f(x_0) + e^{-t}(f(x) - f(x_0)), \\ \gamma_x(0) = x, \quad \gamma_x(t) \in [0, a). \end{cases}$$

It is clear that

$$\gamma'_x(t) = -[f'(\gamma_x(t))]^{-1} e^{-t}(f(x) - f(x_0)).$$

Let  $v = f'(\gamma_x(t))^{-1}(y - f(x_0))$ . By the assumption (3), we have  $D_V^+ k(x) \leq h(k(x))$  and the maximum solution  $r(t)$  of (2.2) is defined on  $[0, a)$  and there exists a sequence  $t_n \rightarrow a$  as  $n \rightarrow \infty$  such that  $\lim_{n \rightarrow \infty} r(t_n) = r^*$  is finite. It follows that  $r(t)$  is continuous on  $[0, a]$  and there is a constant  $M_1$  such that  $|r(t)| \leq M_1, t \in [0, a]$ . By Theorem 2.1, we have

$$\|k(\gamma_x(t))\| \leq |r(t)| \leq M_1, \quad t \in [0, a]$$

as well as  $\|\gamma_x(t)\| \leq M_2, t \in [0, a)$  for some constant  $M_2$ , since  $k$  is coercive, and let

$$M_3 = \sup_{\|x\| \leq M_2} \|f'(x)^{-1}\| < +\infty, \quad x \in D.$$

Let  $t = -h, t \in (a, 0], h \in [0, -a), a < 0$ , for  $t_1, t_2 \in (a, 0]$ , we have

$$\begin{aligned} \|\gamma_x(t_1) - \gamma_x(t_2)\| &= \|\gamma_x(-h_1) - \gamma_x(-h_2)\| = \int_{-h_2}^{-h_1} \|\gamma'_x(s)\| ds \\ &= \int_{-h_2}^{-h_1} \|f'(\gamma_x(s))^{-1}\| e^{-s} \|f(x) - f(x_0)\| ds \\ &\leq \int_{h_1}^{h_2} \|f'(\gamma_x(-u))\| e^u \|f(x) - f(x_0)\| du \\ &\leq M_3 \int_{h_1}^{h_2} \|f(x) - f(x_0)\| e^u du \\ &\leq M_3 e^{-a} \|f(x) - f(x_0)\| |t_1 - t_2|. \end{aligned}$$

So  $\gamma_x(t)$  is Lipschitz continuous on  $(a, 0]$ ,  $\gamma_x(t)$  can also be extended to  $-\infty$ , and Theorem 2.2 is proved.  $\square$

We find Zampieri's results (Theorem 1.3) as a consequence of Theorem 2.2.

**Corollary 2.1** *The mapping  $f \in C^1(E; F)$  is a global diffeomorphism onto  $F$  if*

- (1)  $f'(x) \in \text{Isom}(E, F) \forall x \in E$ ;
- (2)  $\sup_{\|x\| \leq r} \|f'(x)^{-1}\| < +\infty \forall r: 0 < r < +\infty$ ;

- (3) *there exists a locally Lipschitzian coercive function  $k : E \rightarrow R^+$  which admits all  $D_v^+ k(x)$ , and it is such that  $\sup\{D_v^+ k(x) : v = f'(x)^{-1}u, x \in E, u \in F, \|u\| = 1\} < +\infty$ .*

*Proof* Let

$$u = \frac{y - f(x_0)}{\|y - f(x_0)\|}.$$

From condition (3) for arbitrary  $x, x_0 \in E, y \in F, v = f'(x)^{-1}(y - f(x_0))$ .

Define

$$D_v^+ k(x) \leq M.$$

Let  $h(t) = M$ , then the condition (3) of Theorem 2.2 is satisfied, and the corollary is proved.  $\square$

### 3 Applications of the main theorems

Consider the fourth order elliptic boundary value problem

$$\begin{cases} \Delta^2 u + f(x, u) = 0, \\ u \in H_I \text{ or } H_{II}, \end{cases} \quad (3.1)$$

where  $f(x, u)$  is measurable in  $x$  for all  $u$  and has continuous partial derivatives in  $u$  for almost all  $x$ .

Alexiades and Elcart [11] studied the existence of solution of the problem (3.1), and here we will prove the existence and uniqueness of solution of the problem (3.1).

We assume that  $G$  is a bounded domain in  $R^n$  with piecewise smooth boundary  $\partial G$  whose principal curvatures are bounded.

Let

$$H = \{u \in W^{2,2} \cap W_0^{1,2}, \Delta^2 u \in L^2(G)\}$$

and note that  $H$  is a Hilbert space with norm  $\|\cdot\|$  given by

$$\|u\|^2 = \int_G (\Delta u)^2 + \int_G (\Delta^2 u)^2, \quad u \in H,$$

where  $W^{2,2} \cap W_0^{1,2}$  (see [12, Chapter 7]) can be shown to consist of  $W^{2,2}$  limits of smooth functions which vanish on the boundary.

We will first consider the differential operator  $L$  defined by

$$Lu = \Delta^2 u + a(x)u, \quad (3.2)$$

where  $a(x)$  are bounded measurable functions defined in  $G$ .

We shall be concerned with the sets of boundary conditions

$$u = \frac{\partial u}{\partial n} = 0, \quad \text{on } \partial G; \quad (I)$$

or

$$u = \Delta u = 0, \quad \text{on } \partial G; \quad (\text{II})$$

and we define

$$H_I = \left\{ u \in H, \frac{\partial u}{\partial n} = 0, x \in \partial G \right\}, \quad H_{II} = \{ u \in H, \Delta u \in W_0^{1,2} \}.$$

Each of these spaces is a closed subspace of  $H$  and the differential operator  $L$  is a bounded linear operator from  $H_I$  or  $H_{II}$  into  $L^2(G)$ .

Denote

$$\Omega = \inf \frac{\int (\Delta u)^2}{\int u^2}, \quad (3.3)$$

where the infimum is taken over  $H_I$  or  $H_{II}$  according to whether (I) or (II) are the boundary conditions in the relevant eigenvalue problem. In fact,  $\Omega$  is the first eigenvalue in the clamped plate ( $\Delta^2 \varphi - \Omega \varphi = 0$ ) problem.

Let

$$m_0 = \inf_{x \in G} a(x), \quad M_0 = \sup_{x \in G} a(x).$$

**Lemma 3.1** *If  $\inf_{x \in G} a(x) > -\Omega$ , then  $L$  is an invertible operator of  $H_I$  as well as of  $H_{II}$  onto  $L^2(G)$  and*

$$\|u\| \leq C \|Lu\|_0, \quad (3.4)$$

where  $C = 2 + \frac{\Omega}{\varepsilon^2} + \frac{2M_0^2}{\varepsilon^2}$  ( $\varepsilon > 0$ ).

*Proof* Since  $\Omega = \inf \frac{\int (\Delta u)^2}{\int u^2} > 0$  is the first eigenvalue of  $\Delta^2 \varphi - \Omega \varphi = 0$  in  $H_I$  ( $H_{II}$ ), it follows that for all  $u \in H_I$  ( $H_{II}$ ), zero is not an eigenvalue of  $\Delta^2 \varphi - \Omega \varphi = 0$ , so for every  $u \in H_I$  ( $H_{II}$ ), the operator  $Lu = \Delta^2 u + a(x)u$  is an invertible operator of  $H_I$  ( $H_{II}$ ) onto  $L^2(G)$ .

For  $u \in H_I$  ( $H_{II}$ ), we have

$$\int (\Delta u)^2 + \int au^2 = \int uLu \leq \frac{\varepsilon}{2} \int u^2 + \frac{1}{2\varepsilon} \int (Lu)^2,$$

for any  $\varepsilon > 0$  and  $-\Omega < m_0$ , it follows that

$$\begin{aligned} \int (\Delta u)^2 + m_0 \int u^2 &\leq \frac{\varepsilon}{2} \int u^2 + \frac{1}{2\varepsilon} \int (Lu)^2, \\ \int (\Delta u)^2 &\leq \left( \frac{\varepsilon}{2} - m_0 \right) \int u^2 + \frac{1}{2\varepsilon} \int (Lu)^2. \end{aligned}$$

From (3.3), we have

$$\int (\Delta u)^2 \leq \left( \frac{\varepsilon}{2\Omega} + 1 \right) \int (\Delta u)^2 + \frac{1}{2\varepsilon} \int (Lu)^2, \quad \int (\Delta u)^2 \leq \frac{\Omega}{\varepsilon^2} \int (Lu)^2$$

and

$$\begin{aligned}\int (\Delta^2 u)^2 &= \int (Lu - au)^2 \\ &\leq 2 \int (Lu)^2 + 2 \int (au)^2 \\ &\leq \left(2 + \frac{2a^2}{\varepsilon^2}\right) \int (Lu)^2 \leq \left(2 + \frac{2M_0^2}{\varepsilon^2}\right) \int (Lu)^2.\end{aligned}$$

We obtain

$$\begin{aligned}\|u\|^2 &= \int (\Delta u)^2 + \int (\Delta^2 u)^2 \\ &\leq \left(2 + \frac{\Omega}{\varepsilon^2} + \frac{2M_0^2}{\varepsilon^2}\right) \int (Lu)^2 \\ &\leq C \|Lu\|_0^2.\end{aligned}$$

Here  $C = 2 + \frac{\Omega}{\varepsilon^2} + \frac{2M_0^2}{\varepsilon^2}$ .

Now we express (3.1) in the form

$$F(u) = \Delta^2 u + f(u, u)$$

and (3.1) is equivalent to the operator equation

$$F(u) = 0, \quad u \in H_I(H_{II}).$$

For any  $u, \varphi \in H_I(H_{II})$ , we have

$$F'(u)(\varphi) = \Delta^2 \varphi + f'_u(x, u)\varphi.$$

□

**Theorem 3.1** Assume that

- (1)  $\inf_{G \times R} f'_u(x, u) > -\Omega$ ;
- (2) there exists a coercive function  $k: R^n \rightarrow R^+$  which admits the directional right derivatives  $D_V^+ k(x)$  for arbitrary  $u, u_0 \in H_I(H_{II})$ ,  $y \in L^2(G)$ ,  $v = F'(u)^{-1}(y - F(u_0))$  and a continuous mapping  $h: R^+ \rightarrow R^+$  such that  $D_V^+ k(u) \leq h(k(u))$  and for any  $r_0 > 0$ , the maximum solution of the initial value problem

$$\begin{cases} r' = h(r), \\ r(0) = r_0 \end{cases}$$

is defined on  $[0, a)$  and there exists a sequence  $\{t_n\}$  such that  $t_n \rightarrow a$  and  $n \rightarrow \infty$

$$\lim_{n \rightarrow \infty} r(t_n) = r^*$$

is finite. Then there is a unique solution of the equation  $F(u) = 0$  in  $H_I(H_{II})$ .

*Proof* The condition  $\inf_{G \times R} f'_u(x, u) > -\Omega$  implies that, for each  $u$ ,  $F'(u)$  is invertible as a linear operator from  $H_I$  ( $H_{II}$ ) onto  $L^2(G)$ . Furthermore, an upper bound for  $\|F'(u)^{-1}\|$  is provided by (3.4) if the coefficient  $a(x)$  is identified with  $f'_u(x, u)$ . In fact,  $C = O(M_0)$ , where  $C$  is the constant in (3.4), implies that

$$\|[F'(u)]^{-1}\| \leq \alpha \sup_G |f'_u(x, u)| + \beta$$

for positive constant  $\alpha, \beta$ .

By the above discussion and condition (2), the assumptions of Theorem 2.2 are satisfied,  $F$  is a homeomorphism of  $H_I$  ( $H_{II}$ ) onto  $L^2(G)$ , and the equation  $F(u) = 0$  has a unique solution.  $\square$

**Corollary 3.1** *Assume that  $f$  satisfies*

(1)  $\inf_{G \times R} f'_u > -\Omega$ ;

(2) *uniformly in  $x$ ,  $\|f'_u\| = \omega(\|u\|)$ , where  $\omega$  is a continuous map satisfying  $\int_a^\infty \frac{dt}{\omega(t)} = \infty$ .*

*Then there is a unique solution of (3.1) in  $H_I$  ( $H_{II}$ ).*

*Proof* We have from Lemma 3.1 and (3.4)

$$\|[F'(u)]^{-1}\| \leq \alpha \sup_G |f'_u(x, u(x))| + \beta$$

for positive constants  $\alpha, \beta$ .

Denote  $\omega_1(t) = \alpha\omega(t) + \beta$ ; clearly,  $\int_a^\infty \frac{dt}{\omega_1(t)} = \infty$ .

Let

$$k(u) = \int_a^{\|u\|} \frac{dt}{\alpha \sup_G |f'_u(x, u)| + \beta} = \int_a^{\|u\|} \frac{dt}{\omega_1(t)},$$

then  $k(u) \rightarrow +\infty$  as  $\|u\| \rightarrow +\infty$ .

For  $v = F'(u)^{-1}(y - F(u_0))$ ,  $u_0 \in H_I$  ( $H_{II}$ ),  $y \in L^2(G)$ , we have

$$\begin{aligned} D_v^+ k(u) &= \lim_{s \rightarrow 0^+} \frac{k(u + sv) - k(u)}{s} \\ &= \frac{1}{\|\alpha \sup_G |f'_u(x, u)| + \beta\|} \lim_{s \rightarrow 0^+} \frac{\|u + sv\| - \|u\|}{s} = \frac{1}{\omega_1(t)} \lim_{s \rightarrow 0^+} \frac{\|u + sv\| - \|u\|}{s} \\ &\leq \frac{1}{\|F'(u)\|} \|v\| = \|y - F(u_0)\| = M. \end{aligned}$$

Let  $h(t) = M$ , the initial value problem

$$\begin{cases} r' = h(r), \\ r(0) = r_0 \end{cases}$$

changes to

$$\begin{cases} r' = M, \\ r(0) = r_0 > 0 \end{cases} \quad (3.5)$$



the solution of equation (3.5) on  $[0, b]$  is  $r(t) = Mt$ . Hence all the conditions of Theorem 3.1 are satisfied. We get the results of this corollary.  $\square$

**Corollary 3.2** (Alexiades and Elcrat) *Assume that  $f$  satisfies*

- (1)  $\inf_{G \times R} f'_u > -\Omega$ ;
- (2)  $\|f'_u\| = O(\|u\|)$ , uniformly in  $x \in G$ .

*Then there is a unique solution of (3.1) in  $H_I$  ( $H_{II}$ ).*

*Proof* Condition (2) implies that  $\int_a^\infty \frac{dt}{\omega(t)} = \infty$  holds, and the result of Elcart and Sigillito in [11] becomes a special case of Theorem 3.1.  $\square$

#### Competing interests

The authors declared that they have no conflicts of interest to this work.

#### Authors' contributions

All authors contributed equally to the manuscript. All authors read and approved the final manuscript.

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